Heat and Mass Transfer in oscillatory flow of a non-Newtonian fluid between two inclined porous plates placed in a Magnetic field

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Abstract-In the present paper a study on heat and mass transfer during the flow of a non-Newtonian fluid between two parallel plates in a magnetic field has been made and after forming the governing equations of the flow and establishing the boundary conditions according to the nature of the flow, the equations have been solved for velocity profile and graphical study has been done with respect to various parameters to verify the results which are very useful in metallic industries.

Keywords: MHD, Oscillatory flow, Thermal Radiation.

1 INTRODUCTION

The flow of visco-elastic fluids through various channels placed in a magnetic field has got importance in many industries of chemical processing, food preservation, petroleum production, polymer production and electro-static precipitation. Such flows occur in these areas as they transfer heat and mass which needs a deep knowledge of MHD flow. Several researchers have contributed their work in this field. Ganesan and Palani (2013) studied the convective flow over an inclined plate with variable heat and mass flux. Nandeppanavar et al. (2008) worked on the heat transfer in MHD visco-elastic boundary layer over a stretching sheet with thermal radiation and non-uniform heat source. Goyal M. and Kumari K. (2013) discussed the heat and mass transfer in MHD oscillatory flow between two inclined porous plates with radiation absorption and chemical reaction.

In the present paper, the concentration exists on non-Newtonian fluids flowing through parallel plate channel placed in a magnetic field. It is assumed that channel is filled with porous medium and is inclined with the axis. Different equations are formed by inserting the necessary terms of various parameters as required by the physical nature of the problem in the standard equation of continuity, momentum, energy and diffusion. They have been solved using non-dimensional parameters and the derived equation between magnetic field and the velocity profile has been used for graphical study and analyzing the result. The result has been found consistent in physical nature of the problem.

2 FORMULATION OF THE PROBLEM

The MHD oscillatory flow of non-Newtonian fluid between two inclined porous plates in the presence of magnetic field is considered subject to thermal radiation, absorption and chemical reaction. Let X-axis be along the lower plate and straight line perpendicular to that as the Y-axis. In this present visco-elastic fluid model, a non-Newtonian parameter has been introduced in the momentum equation and its study has been conducted with respect to transversely applied magnetic field. Under such assumptions the momentum equation, energy equation and diffusion equation which govern the flow field are:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + v_1\frac{\partial^2 u'}{\partial y'^2} + v_2\frac{\partial^3 u'}{\partial y'^2\partial t'} - \frac{v_1}{K'}u' - \frac{\sigma_e {B_o}^2 u'}{(1+W^2)} + g\beta_T(T' - {T_o}')sin\phi + g\beta_c(C' - {C_o}')sin\phi \quad (1)$$

$$\rho C_P\frac{\partial T'}{\partial t'} = K\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q}{\partial y'} - Q(T' - {T_o}') + Q_c(C' - {C_o}') \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r(C' - {C_o}') \quad (3)$$

The boundary conditions become:

$$u' = 0, T' = {T_w}', C' = {C_w}' \; at \; y = 1$$

$$u' = 0, T' = {T_o}', C' = {C_o}' \; at \; y = 0 \quad (4)$$

The heat flux is given by:

$$\frac{\partial q}{\partial y'} = 4\alpha^2(T' - {T_o}') \quad (5)$$

Non-dimensional variables are:

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$$x = \frac{x'}{a}, y = \frac{y'}{a}, u = \frac{u'}{U}, Re = \frac{Ua}{u_1}, \theta = \frac{(T' - T_o')}{(T'_w - T_o')},$$

$$C = \frac{(C' - C_o')}{(C'_w - C_o')}, Gc = \frac{g\beta_c(C'_w - C_o')a^2}{u_1 U}, Gr = \frac{g\beta_T(T'_w - T_o')a^2}{u_1 U},$$

$$P = \frac{aP'}{\rho u_1 U}, Da = \frac{K'}{a^2}, H^2 = \frac{a^2\sigma_e B_0^2}{\rho v_1(1+W^2)}, Q = \frac{Q_C(C'_w - C_o')a^2}{k(T'_w - T_o')}$$

$$Pe = \frac{Ua\rho C_p}{k}, R^2 = \frac{4\alpha^2 a^2}{k}, Sc = \frac{D}{aU}, S^2 = \frac{1}{Da},$$

$$E = \frac{Qa^2}{k}, J = \frac{K_\rho a}{U} \qquad (6)$$

In view of above non dimensional variables, the system of the equation (1) – (3) reduce to the following dimensionless form

$$Re\frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \gamma\frac{\partial^3 u}{\partial y^2} - \left(s^2 + \frac{H^2}{(1+W^2)}\right)u + Gr_1\theta + Gc_1 C \qquad (7)$$

$$P_e\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} + (R^2 - E)\theta + Q_1 \qquad (8)$$

$$\frac{\partial C}{\partial t} = Sc\frac{\partial^2 c}{\partial y^2} - JC \qquad (9)$$

Where $Gr_1 = Grsin\phi \text{ and } Gc_1 = Gcsin\phi$

The corresponding boundary conditions in dimensionless form are:

$$u = 0, \theta = 1, C = 1 \text{ at } y = 1$$

$$u = 0, \theta = 0, C = 0 \text{ at } y = 0 \qquad (10)$$

2.1 SOLUTION OF THE PROBLEM

In order to solve equations (7) – (10) for purely oscillatory flow, let

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, u(y,t) = u_o(y)e^{i\omega t}, \theta(y,t) = \theta_o(y)e^{i\omega t},$$
$$C(y,t) = C_o(y)e^{i\omega t} \qquad (11)$$

Substituting in equation (11) into equations (7) - (10), we obtain

$$(1 + i\gamma\omega)\frac{d^2u_o}{dy^2} - \frac{{M_2}^2}{(1+W^2)}u_o = -\lambda - Gr_1\theta_0 - Gc_1C_0 \qquad (12)$$

$$\frac{d^2\theta_o}{dy^2} + {M_1}^2\theta_o + Q_1C_0 = 0 \qquad (13)$$

$$\frac{d^2C_o}{dy^2} - {M_3}^2C_o = 0 \qquad (14)$$

Where,

$$L^2 = 1 + i\omega\gamma$$

$$M_1^2 = R^2 - E - i\omega P_e$$

$$M_2^2 = S^2 + \frac{H^2}{(1+W^2)} + i\omega Re$$

$$M_3^2 = \frac{J + i\omega}{Sc}$$

$$k_1 = \frac{Q_1}{M_1^2 + M_3^2}$$

The corresponding boundary conditions are:

$$u_o = 0, \theta_o = 1, C_o = 1 \text{ at } y = 1$$

$$u_o = 0, \theta_o = 0, C_o = 0 \text{ at } y = 0 \qquad (15)$$

Solving equations (12) to (14) under the boundary conditions (15), we get the solution of the velocity, temperature and concentration distributions as follows:

$$u(y,t) = \left[\begin{array}{c} Gr_1\left\{\begin{array}{c}\left(\frac{1+k_1}{{M_1}^2L^2+\frac{{M_2}^2}{(1+W^2)}}\right)\frac{\sin M_1y}{\sin M_1} + \left(\frac{k_1}{{M_3}^2L^2+\frac{{M_2}^2}{(1+W^2)}}\right)\frac{\sin M_3y}{\sin M_3} - \\ \left(\frac{1+k_1}{{M_1}^2L^2+\frac{{M_2}^2}{(1+W^2)}} + \frac{k_1}{{M_3}^2L^2+\frac{{M_2}^2}{(1+W^2)}}\right)\frac{\sin\frac{M_2}{L\sqrt{(1+W^2)}}y}{\sin\frac{M_2}{L\sqrt{(1+W^2)}}}\end{array}\right\} + \\ \frac{Gc_1}{{M_3}^2L^2-\frac{{M_2}^2}{(1+W^2)}}\left(\frac{\sinh\frac{M_2}{L\sqrt{(1+W^2)}}y}{\sinh\frac{M_2}{L\sqrt{(1+W^2)}}} - \frac{\sin M_3y}{\sin M_3}\right) \\ +\frac{\lambda}{\frac{{M_2}^2}{(1+W^2)}}\left(1 - \cosh\frac{M_2}{L\sqrt{(1+W^2)}}y\right)\left(1 - \frac{\sinh\frac{M_2}{L\sqrt{(1+W^2)}}y}{\sinh\frac{M_2}{L\sqrt{(1+W^2)}}}\right)\end{array}\right]e^{i\omega t}$$

(16)

$$\theta(y,t) = \left[\left(1 + \frac{Q_1}{{M_1}^2+{M_3}^2}\right)\frac{\sin M_1y}{\sin M_1} - \left(\frac{Q_1}{{M_1}^2+{M_3}^2}\right)\frac{\sin M_3y}{\sin M_3}\right]e^{i\omega t} \qquad (17)$$

$$C(y,t) = \left(\frac{\sin M_3y}{\sin M_3}\right)e^{i\omega t} \qquad (18)$$

To observe the velocity profile with respect to applied magnetic field, suitable values for different parameters are chosen to plot the curve between Hartmann Number

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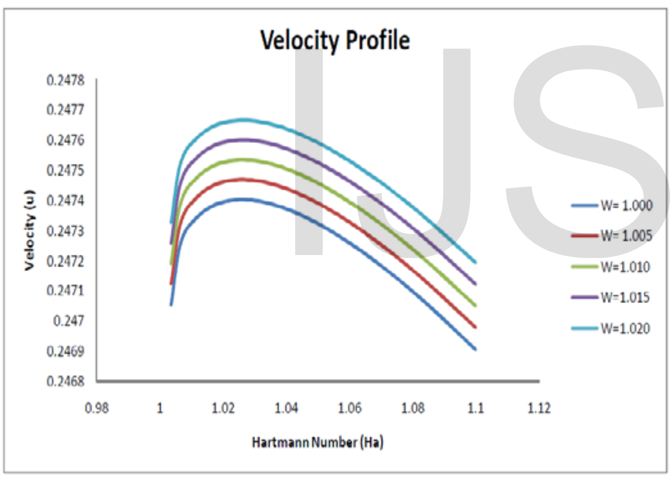
and velocity profile for different values of non-Newtonian factors.

2.3 RESULT AND DISCUSSION

TABLE: Hartmann Number and Velocity Profile

Hartmann Number (Ha)	Velocity Profile				
	Non-Newtonian factor (W=1.000)	Non-Newtonian factor (W=1.005)	Non-Newtonian factor (W=1.010)	Non-Newtonian factor (W=1.015)	Non-Newtonian factor (W=1.020)
1.003510827	0.247054850	0.247123255	0.247191350	0.247259135	0.247326609
1.006565190	0.247259395	0.247326233	0.247392771	0.247459010	0.247524949
1.011635702	0.247342916	0.247409214	0.247475214	0.247540917	0.247606321
1.018692831	0.247390623	0.247456726	0.247522529	0.247588035	0.247653241
1.027696397	0.247402118	0.247468377	0.247534333	0.247599985	0.247665334
1.038596659	0.247377040	0.247443812	0.247510273	0.247576423	0.247642262
1.051335585	0.247315068	0.247382714	0.247450039	0.247517041	0.247583721
1.065848246	0.247215922	0.247284809	0.247353359	0.247421571	0.247489446
1.082064266	0.247079373	0.247149867	0.247220006	0.247289789	0.247359217
1.099909257	0.246905237	0.246977709	0.247049803	0.247121519	0.247192858



The graphical study indicates that the velocity increases in the beginning obtaining a maximum value, it starts decreasing for each value of non-Newtonian factor but somehow the velocity profile shows a resonance character, the curvature is highly dominated by the non-Newtonian factor. Also the derived relation for the velocity has a non-Newtonian factor as an important component in controlling the velocity. However the presence of other terms cannot be ignored. The applied magnetic field creates a resistive force due to which the particle loose the velocity, the presence of trigonometric function bearing non-Newtonian terms put the motion in hyperbolic nature and for a certain value of magnetic field the resonance occurs. Therefore this value can be used in controlling the metallic flow in industries by applying the magnetic field.

3 NOMENCLATURE

Symbol	Quantity
T'	Fluid temperature
u'	Axial velocity
U	Flow mean velocity
R_e	Reynolds number
P_e	Peclet number
S_c	Schmidt number
G_r	Grashof number
G_c	Solutal Grashof number
C_p	Specific heat at constant pressure
Q_1	Radiation absorption parameter
$B_\circ$	Magnetic field
k	Thermal conductivity
E	Heat source parameter
R	Radiation parameter
H	Hartmann number
J	Chemical reaction parameter
S	Porous medium shape factor
θ	Fluid temperature
W	non-Newtonian parameter
$T_\circ, T_w$	Walls of temperature
α	Mean radiation absorption coefficient

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